

Lack of a Minimal Pair for Generic Computability

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In a recent paper, Jockusch and Schupp introduce and analyze the notion of generic computation. A generic computation of a real is a partial recursive function which correctly computes most of the bits of the real, but which may diverge on some inputs, provided that these inputs have an asymptotic density of 0 in the natural numbers. It turns out that this notion of computation has many properties that are rather counterintuitive from a recursion theoretic point of view.

We present our result that there are no minimal pairs for generic computation, in the sense that for any nonrecursive reals A and B , there is a real C , which is not generically computable, but such that either A or B is able to generically compute C . Downey, Jockusch, and Schupp proved this result in the case where A and B are both Δ_2^0 , and indeed this distinction appears to be significant, in that our technique requires a nonuniformity in the algorithm, based on whether one, both, or neither of the reals is Δ_2^0 .