UNDECIDABILITY AND DEFINITIONS OF INTEGERS AND INTEGRAL FUNCTIONS

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ABSTRACT. Let M be an algebraic (possibly infinite) extension of $\mathbb{Q}$ or a rational function field over a finite field of constants. Let R_M be the ring of integers or integral functions of M, i.e. the set of elements of M that are roots of monic irreducible polynomials with coefficients in $\mathbb{Z}$ or a polynomial ring over a finite field. We investigate the following question. Can R_M be defined over M by a formula of the form

$$\{t \in M : Q_1x_1 \ldots Q_nx_nP(t, x_1, \ldots, x_m) = 0\}$$

where each $Q_1, \ldots, Q_n$ is either an existential or a universal quantifier and $P(t, x_1, \ldots, x_n)$ is a polynomial with coefficients in M? Further, if such a formula exists, what is the smallest number of universal quantifiers necessary and what are the implications for the first-order undecidability of the field M?

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