Polynomial-time Set Recursion

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A central notion of computational complexity theory is that of polynomial-time computation on finite strings. Polynomial-time computation on infinite strings has been considered by a number of authors (Deolalikar, Hamkins, Schindler, Welch and perhaps others) in the context of infinite-time Turing machines. But what should polynomial-time computation mean for sets in general? I'll present such a definition, based on a natural generalisation of the Bellantoni-Cook schemes characterising polynomial-time computation on finite strings in terms of "safe recursion". One can also provide a machine model for this theory, in which processors attached to each set run in parallel. Finally, we ask what complexity classes on finite strings arise under various interpretations of finite strings as sets; one natural such interpretation leads to the class defined by alternating exponential time with polynomially-many alternations. A similar class arose earlier as the complexity of the theory of the real numbers as an ordered additive group. This is joint work with Arnold Beckmann and Sam Buss.