From causality to computability: a quantum version of Gandy's theorem

Pablo Arrighi[1,2]

[1] École normale supérieure de Lyon, LIP, 46 allée d'Italie, 69008 Lyon, France
[2] Université de Grenoble, LIG, 220 rue de la chimie, 38400 SMH, France
parrighi@imag.fr

Abstract. Several works have shown how quantum evolutions, if one follows the postulates of quantum theory 'by-the-book', could breach the physical Church-Turing thesis [8, 7]. On the other hand, we have shown in [4] that if a quantum evolutions is causal (i.e. bounded speed of information), then it is also localizable (i.e. described by a circuit of local gates). The same property does not hold true in the probabilistic setting [3], as we shall explain. Hence in the quantum setting, causality alone entails that evolutions take a very operational form, suggesting that this additional physical hypothesis suffices to prove the physical Church-Turing thesis. I will then explain a recent result [2] that draws a clear line as to when this is the case. The approach is clearly inspired by Gandy [6], as it formulates some postulates about physics, such as homogeneity of space and time, bounded density and velocity of information — and then derives the physical Church-Turing thesis is a consequence.

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