Incomputability of Simply Connected Planar Continua

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Every nonempty $\Sigma^0_1$ set in $\mathbb{R}^n$ contains a computable point.
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Not every nonempty $\Pi^0_1$ set in $\mathbb{R}^n$ contains a computable point.
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Not every nonempty $\Pi^0_1$ set in $\mathbb{R}^n$ contains a computable point.

If a nonempty $\Pi^0_1$ subset $F \subseteq \mathbb{R}^1$ contains no computable points, then $F$ must be disconnected.
Introduction

Main Theorem

- Every nonempty $\Sigma^0_1$ set in $\mathbb{R}^n$ contains a computable point.
- Not every nonempty $\Pi^0_1$ set in $\mathbb{R}^n$ contains a computable point.
- If a nonempty $\Pi^0_1$ subset $F \subseteq \mathbb{R}^1$ contains no computable points, then $F$ must be *disconnected*.
- Does there exist a nonempty (simply) *connected* $\Pi^0_1$ set in $\mathbb{R}^n$ without computable points?

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Every nonempty $\Sigma^0_1$ set in $\mathbb{R}^n$ contains a computable point.

Not every nonempty $\Pi^0_1$ set in $\mathbb{R}^n$ contains a computable point.

If a nonempty $\Pi^0_1$ subset $F \subseteq \mathbb{R}^1$ contains no computable points, then $F$ must be disconnected.

Does there exist a nonempty (simply) connected $\Pi^0_1$ set in $\mathbb{R}^n$ without computable points?

The main theme of this talk is Computability Theory for Connected Spaces.
{\mathcal{B}_e}_{e \in \mathbb{N}}: an effective enumeration of all rational open balls.

1. $x \in \mathbb{R}^n$ is \textit{computable} if $\{e \in \mathbb{N} : x \in \mathcal{B}_e\}$ is c.e.

Equivalently, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is computable iff $x_i$ is computable for each $i \leq n$. 

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Definition

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   Equivalently, $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is computable iff $x_i$ is computable for each $i \leq n$.

2. $F \subseteq \mathbb{R}^n$ is $\Pi^0_1$ (or **co-c.e. closed**) if $F = \mathbb{R}^n \setminus \bigcup_{e \in W} B_e$ for a c.e. set $W$. 

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1. Not every nonempty \(\Pi^0_1\) set in \(\mathbb{R}^1\) contains a computable point.
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---

1. Not every nonempty $\Pi^0_1$ set in $\mathbb{R}^1$ contains a computable point.

2. (*Category*) Every nonempty *co-meager* $\Pi^0_1$ set in $\mathbb{R}^n$ contains a computable point.

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Incomputability of Simply Connected Planar Continua
Definition

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1. Not every nonempty \( \Pi^0_1 \) set in \( \mathbb{R}^1 \) contains a computable point.

2. (Category) Every nonempty **co-meager** \( \Pi^0_1 \) set in \( \mathbb{R}^n \) contains a computable point.

3. (Measure) Not every nonempty **positive measure** \( \Pi^0_1 \) set in \( \mathbb{R}^1 \) contains a computable point.
Definition

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2. *(Category)* Every nonempty co-meager \( \Pi^0_1 \) set in \( \mathbb{R}^n \) contains a computable point.

3. *(Measure)* Not every nonempty positive measure \( \Pi^0_1 \) set in \( \mathbb{R}^1 \) contains a computable point.

4. *(Connectedness)* What about **connected**, **simply connected**, or **contractible** \( \Pi^0_1 \) sets?
Connected $\prod^0_1$ Sets

Observation

1. Every nonempty connected $\prod^0_1$ subset $P \subseteq \mathbb{R}^1$ contains a computable point.
Connected $\Pi^0_1$ Sets

Observation

1. Every nonempty connected $\Pi^0_1$ subset $P \subseteq \mathbb{R}^1$ contains a computable point.

Fact

1. There exists a nonempty connected $\Pi^0_1$ subset $P^{(2)} \subseteq \mathbb{R}^2$ without computable points.

2. There exists a nonempty simply connected $\Pi^0_1$ subset $P^{(3)} \subseteq \mathbb{R}^3$ without computable points.
Example

\[ P^{(2)} \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]

\[ \vdots \]

\[ P \]

\[ P^{(n)} = \bigcup_{k<n} ([0, 1]^k \times P \times [0, 1]^{n-k-1}) \text{ for } P \subseteq [0, 1]. \]
Example

\[ P^{(n)} = \bigcup_{k<n} ([0, 1]^k \times P \times [0, 1]^{n-k-1}) \text{ for } P \subseteq [0, 1]. \]

Let \( P \subseteq [0, 1] \) be a \( \Pi^0_1 \) set without computable points.
Let $P \subseteq [0, 1]$ be a $\Pi^0_1$ set without computable points.

$P^{(2)} \subseteq [0, 1]^2$ is a connected $\Pi^0_1$ set without computable points.
\[ P^{(n)} = \bigcup_{k<n}([0,1]^k \times P \times [0,1]^{n-k-1}) \text{ for } P \subseteq [0,1]. \]

- Let \( P \subseteq [0,1] \) be a \( \Pi_1^0 \) set without computable points.
- \( P^{(2)} \subseteq [0,1]^2 \) is a connected \( \Pi_1^0 \) set without computable points.
- \( P^{(3)} \subseteq [0,1]^3 \) is a simply connected \( \Pi_1^0 \) set without computable points.
**Introduction**

**Main Theorem**

- **X** is \( n \)-connected \( \iff \) the first \( n + 1 \) homotopy groups vanish identically.
- **X** is path-connected \( \iff \) **X** is 0-connected.
- **X** is simply connected \( \iff \) **X** is 1-connected.
- **X** is contractible \( \iff \) the identity map on **X** is null-homotopic.
- **X** is contractible \( \implies \) **X** is \( n \)-connected for any \( n \).
\( X \) is \( n \)-connected \( \iff \) the first \( n + 1 \) homotopy groups vanish identically.

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Observation

Let \( P \subseteq [0, 1] \) be a \( \Pi^0_1 \) set without computable points.
**Introduction**

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- X is $n$-connected $\iff$ the first $n + 1$ homotopy groups vanish identically.
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- X is contractible $\implies$ X is $n$-connected for any $n$.

**Observation**

- Let $P \subseteq [0, 1]$ be a $\Pi^0_1$ set without computable points.
- $P^{(n+2)} \subseteq [0, 1]^{n+2}$ is $n$-connected, but not $n + 1$-connected.
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- $X$ is \textit{n}-connected $\iff$ the first $n + 1$ homotopy groups vanish identically.
- $X$ is \textit{path}-connected $\iff$ $X$ is \textit{0}-connected.
- $X$ is \textit{simply connected} $\iff$ $X$ is \textit{1}-connected.
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- Let $P \subseteq [0, 1]$ be a $\Pi_1^0$ set without computable points.
- $P^{(n+2)} \subseteq [0, 1]^{n+2}$ is $n$-connected, but not $n + 1$-connected.
- $P^{(n)}$ is not contractible for any $n$. 

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**Incomputability of Simply Connected Planar Continua**
Observation (Restated)

- Not every nonempty $n$-connected $\Pi^0_1$ set in $\mathbb{R}^{n+2}$ contains a computable point, for any $n \in \mathbb{N}$.
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- Every nonempty $n$-connected $\Pi^0_1$ set in $\mathbb{R}^{n+1}$ contains a computable point, for $n = 0$. 

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- Every nonempty $n$-connected $\Pi^0_1$ set in $\mathbb{R}^{n+1}$ contains a computable point, for $n = 0$.

Question

1. (Le Roux-Ziegler) Does every simply connected planar $\Pi^0_1$ set contain a computable point?
2. Does every contractible Euclidean $\Pi^0_1$ set contain a computable point?
Observation (Restated)

- Not every nonempty \( n \)-connected \( \mathcal{P}_1^0 \) set in \( \mathbb{R}^{n+2} \) contains a computable point, for any \( n \in \mathbb{N} \).
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Main Theorem

Not every nonempty contractible planar \( \mathcal{P}_1^0 \) set contains a computable point.
Main Theorem

Not every nonempty contractible planar $\Pi^0_1$ set contains a computable point.
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Not every nonempty contractible planar $\Pi^0_1$ set contains a computable point.

Proof Idea

- Let $P \subseteq \mathbb{R}$ be a $\Pi^0_1$ set without computable points.
Main Theorem

Not every nonempty contractible planar $\mathcal{P}_1^0$ set contains a computable point.

Proof Idea

- Let $P \subseteq \mathbb{R}$ be a $\mathcal{P}_1^0$ set without computable points.
- Stretch $[0, 1] \times P$ along a stray snake $A$ whose destination is a fixed incomputable point (i.e., $A$ is Miller’s computable arc whose end-point is an incomputable left-c.e. real).
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- All path-component of $[0, 1] \times P$ will be bundled at the destination end-point.
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- All path-component of $[0, 1] \times P$ will be bundled at the destination end-point.
- Thus, the desired $\Pi^0_1$ set will be homeomorphic to the Cantor fan.
Proof Idea

Stretch $[0, 1] \times P$ along a *stray snake* $A$.

A snake $A$

Destination of the snake (incomputable point)

$([0, 1] \times P)$

(Cantor fan)

The desired $\Pi_1^0$ set $D$ will be homeomorphic to the Cantor fan.
A fat approximation of Cantor set:

A construction of Cantor set

Fat approx. of Cantor set

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$P$: a $\Pi^0_1$ subset of Cantor set.
$P_s$: a fat approximation of $P$ at stage $s$.
$I_s, r_s$: the leftmost and rightmost of $P_s$. 
A fat approximation of Cantor set:

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- \( P \): a \( \Pi^0_1 \) subset of Cantor set.
- \( P_s \): a fat approximation of \( P \) at stage \( s \).
- \( l_s, r_s \): the leftmost and rightmost of \( P_s \).
- \( [l_s, l_{s+1}] \cap P_s, [r_{s+1}, r_s] \cap P_s \) contains intervals \( l^l_s, l^r_s \).
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- $P$: a $\Pi^0_1$ subset of Cantor set.
- $P_s$: a fat approximation of $P$ at stage $s$.
- $l_s, r_s$: the leftmost and rightmost of $P_s$.
- $[l_s, l_{s+1}] \cap P_s, [r_{s+1}, r_s] \cap P_s$ contains intervals $I^l_s, I^r_s$.
- We call these intervals $I^l_s, I^r_s \subseteq P_s \setminus P_{s+1}$ free blocks.
Prepare a stretched $\Pi^0_1$ class $D_0^- = P \times [0, 1]$.

- $P \subseteq \mathbb{R}^1$: a $\Pi^0_1$ set without computable points.
- $P_s$: a fat approximation of $P$ (Note that $P = \bigcap_s P_s$).
- $D_0^- = [0, 1] \times P_0$. 
\( D_0 \) is the following connected closed set.

\[
\begin{array}{c}
\text{Free block} \\
\text{Body} \\
\text{Free block}
\end{array}
\]

The desired \( \Pi^0_1 \) set \( D \) will be obtained by carving \( D_0 \).
There is a computable sequence \( \{J_s\} \) of rational open intervals s.t.

- \( \min J_s \to \alpha \) as \( s \to \infty \).
- \( \text{diam}(J_s) \to 0 \) as \( s \to \infty \).
- Either \( J_{s+1} \subset J_s \) or \( \max J_s < \min J_{s+1} \), for each \( s \).
Our construction starts with $D_0$. 

Stretched

Free block

Body

Free block
By carving free blocks, stretch $P_0$ toward $\max J_0$. 
By carving free blocks, stretch $P_0$ toward $\min J_0$. 

\[ \min J_0 \]
Proceed one step with a fat approximation of $P$. 

$$\min J_0 \quad \max J_0$$
$D_1$ is defined by this,
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- If $J_1 \subset J_0$, then the construction of $D_2$ is similar as that of $D_1$.
- i.e., on the top block, stretch toward $\max J_1$ and back to $\min J_1$, by caving free blocks.
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- In general, similar for $J_{s+1} \subset J_s$. 
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In general, similar for $J_{s+1} \subset J_s$.

Only the problem is the case of $J_{s+1} \not\subset J_s$!
In the case of $J_{s+1} \notin J_s$:

Overview of $D_s$ (above $D_p$)

Pick the greatest $p \leq s$ such that $J_{s+1} \subset J_p$. 

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In the case of $J_{s+1} \not\subset J_s$:

Overview of $D_s$ (above $D_p$)

Go back to $D_p$ by caving free blocks into the shape of $P$. 
Overview of $D_s$ (above $D_p$)

By caving free blocks on $D_p$ into the shape of $P$, stretch toward $\max J_{s+1}$ and back to $\min J_{s+1}$. 
Main Theorem (Restated)

Not every nonempty contractible planar \( \Pi_1^0 \) set contains a computable point.

\[ D = \bigcap_s D_s \text{ is } \Pi_1^0. \]
Main Theorem (Restated)

Not every nonempty contractible planar $\Pi_1^0$ set contains a computable point.

- $D = \bigcap_s D_s$ is $\Pi_1^0$.
- $D$ is obtained by bundling $[0, 1] \times P$ at $(\alpha, y) \in \mathbb{R}^2$ for some $y$. 
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Not every nonempty contractible planar $\Pi_1^0$ set contains a computable point.

- $D = \bigcap_s D_s$ is $\Pi_1^0$.
- $D$ is obtained by bundling $[0, 1] \times P$ at $(\alpha, y) \in \mathbb{R}^2$ for some $y$.
- $D$ is path-connected by the property of an approximation $\{J_s\}$ of the incomputable left-c.e. real $\alpha$.
Main Theorem (Restated)

Not every nonempty contractible planar $\mathcal{N}_1^0$ set contains a computable point.

- $D = \cap_s D_s$ is $\mathcal{N}_1^0$.
- $D$ is obtained by bundling $[0, 1] \times P$ at $(\alpha, y) \in \mathbb{R}^2$ for some $y$.
- $D$ is path-connected by the property of an approximation $\{J_s\}$ of the incomputable left-c.e. real $\alpha$.
- Therefore, $D$ is homeomorphic to Cantor fan, and contractible.
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Not every nonempty contractible planar $\Pi_1^0$ set contains a computable point.

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- $D$ is path-connected by the property of an approximation $\{J_s\}$ of the incomputable left-c.e. real $\alpha$.
- Therefore, $D$ is homeomorphic to Cantor fan, and contractible.
- Stretching $[0, 1] \times P$ cannot introduce new computable points.
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Not every nonempty contractible planar \( \mathcal{N}_1 \) set contains a computable point.

- \( D = \bigcap_s D_s \) is \( \mathcal{N}_1^0 \).
- \( D \) is obtained by bundling \([0, 1] \times P\) at \((\alpha, y) \in \mathbb{R}^2\) for some \( y \).
- \( D \) is path-connected by the property of an approximation \( \{J_s\} \) of the incomputable left-c.e. real \( \alpha \).
- Therefore, \( D \) is homeomorphic to Cantor fan, and contractible.
- Stretching \([0, 1] \times P\) cannot introduce new computable points.
- Of course, \((\alpha, y)\) is also incomputable.
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- $D$ is obtained by bundling $[0, 1] \times P$ at $(\alpha, y) \in \mathbb{R}^2$ for some $y$.
- $D$ is path-connected by the property of an approximation $\{J_s\}$ of the incomputable left-c.e. real $\alpha$.
- Therefore, $D$ is homeomorphic to Cantor fan, and contractible.
- Stretching $[0, 1] \times P$ cannot introduce new computable points.
- Of course, $(\alpha, y)$ is also incomputable.
- Hence, $D$ has no computable points.
Corollary

For every \( \Pi^0_1 \) class \( P \), there is a contractible planar \( \Pi^0_1 \) set \( D \) such that \( D \) is Turing-degree-isomorphic to \( P \).
Corollary
For every $\Pi^0_1$ class $P$, there is a contractible planar $\Pi^0_1$ set $D$ such that $D$ is Turing-degree-isomorphic to $P$.

Definition ($\text{RCA}_0$)
A sequence $(B_i)_{i \in \mathbb{N}}$ of open rational balls is disk-like if $\bigcup_{i < n} B_i$ is homeomorphic to $(0, 1)^2$ for any $n \in \mathbb{N}$.

Corollary
The following are equivalent over $\text{RCA}_0$:
- **WKL$_0$**: Every infinite tree has a path;
- **Heine-Borel**: Every covering of $[0, 1]$ has a finite subcovering.
- **Heine-Borel(Disk)**: Every disk-like covering of $[0, 1]^2$ has a finite subcovering.
Question

- What about Medvedev degrees of contractible planar $\Pi_1^0$ sets?
- Does every nonempty locally connected planar $\Pi_1^0$ set contain a computable point?
Thank you!