Phase transitions related to the pigeonhole principle

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Joint work with Andreas Weiermann

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Gödel's first incompleteness theorem (1931)

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Different examples, different theories

Goodstein's theorem, Finite Kruskal's Theorem, H. Friedman's BRT and other combinatorial statements, etc.

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Our approach: densities

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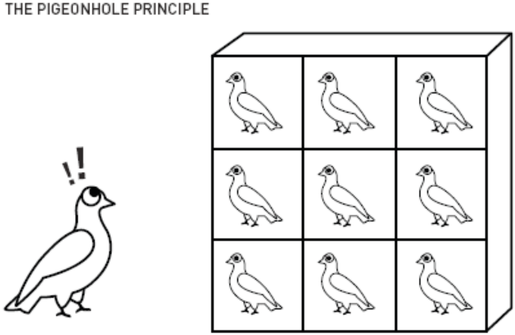
Our approach: densities

J. Paris introduced densities in the late 70's to obtain independence results.


THE PIGEONHOLE PRINCIPLE
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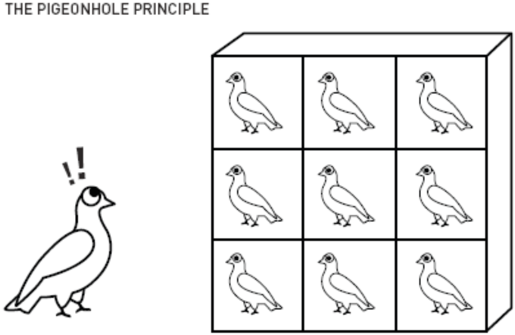

Dirichlet (1834, *Schubfachprinzip*)

If $n+1$ pigeons have been put into n pigeonholes, then at least one pigeonhole will contain more than 1 pigeon.

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- (Infinite) Ramsey's Theorem (1930): $\forall n \, \forall k \, \mathrm{RT}^n_k$
- The pigeonhole principle (PHP): finite version of RT^1_k
- (J. Hirst, 1987) For any natural number k,

$$\mathrm{RCA}_0 \vdash \mathrm{RT}^1_k,$$

whereas

$$\mathrm{RCA}_0 \nvdash \forall k \, \mathrm{RT}^1_k.$$

(even $\mathrm{WKL}_0 \nvdash \forall k \, \mathrm{RT}^1_k$)

Densities related to the pigeonhole principle

- We study RT^1_2 and $\forall k\ \mathrm{RT}^1_k$.

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- We study RT^1_2 and $\forall k\ \mathrm{RT}^1_k$.
- Open problem on Ramsey's theorem for pairs and two colors (RT^2_2) in reverse mathematics.
    - Simpson [1998]: $n \geq 3$ and $k \geq 2$:

$$\mathrm{RCA}_0 \vdash \mathrm{RT}^n_k \leftrightarrow \mathrm{ACA}_0$$

    - Seetapun and Slaman [1995]:

$$\mathrm{RCA}_0 \vdash \mathrm{RT}^2_2 \not\leftrightarrow \mathrm{ACA}_0$$

Strength of RT^2_2?

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- If $f\colon \mathbb{N} \to \mathbb{N}$ is increasing and unbounded, then

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- $A_d = d$th approximation/branch of the Ackermann function A_ω.

Theorems

Theorem 1 - Lower bound

Let $d \in \mathbb{N}$. If $f(i) = i^{\frac{1}{A_d^{-1}(i)}}$, then

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Theorem 2 - Upper bound

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Claim: Let $Y \subseteq [a, b]$. If $|Y| > 2^{A_d(a2^{k+1})2^k}$, then Y is k-PHP-dense(f).
(Proof by induction on k.) □

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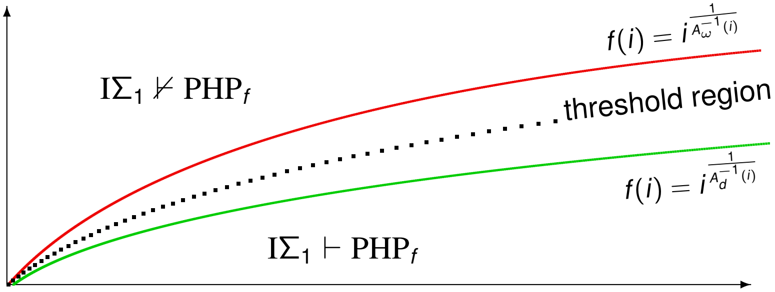

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(Proof by induction on k.) □

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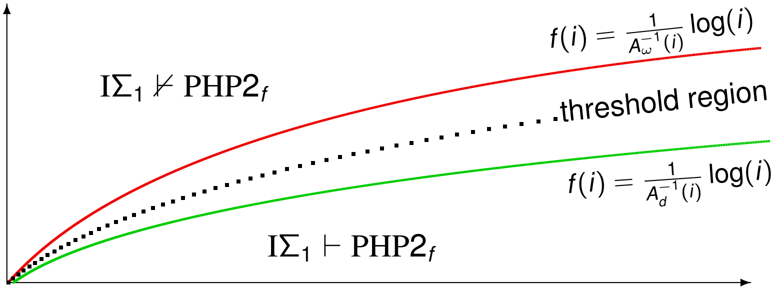

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Ordinals

- α-PHP2-density for $\alpha > \omega^2$.
- Other notions of density closely related to RT^n_k involving ordinals.

Thanks - Personal Information

Thank you for listening!

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