CiE, Sofia, June 2011.

Applying causality principles to the axiomatization of Probabilistic Cellular Automata

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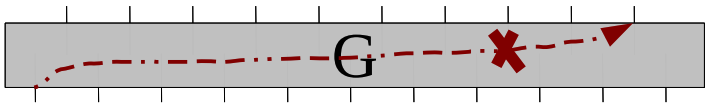
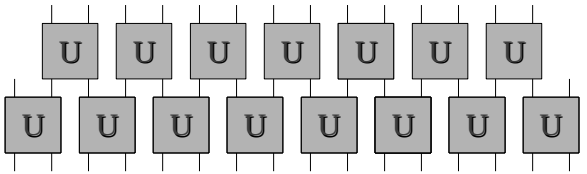
Examples

- Unitary ⟷ Structure theorem / Representation theorem ⟷
  - Spectral form
  - Q. circuit
- TPCP ⟷ Characterization theorem / Axiomatization, test ⟷
  - Gen. Meas.
  - Kraus rep.

Interested in

- Causality ⟷
  - Localizability

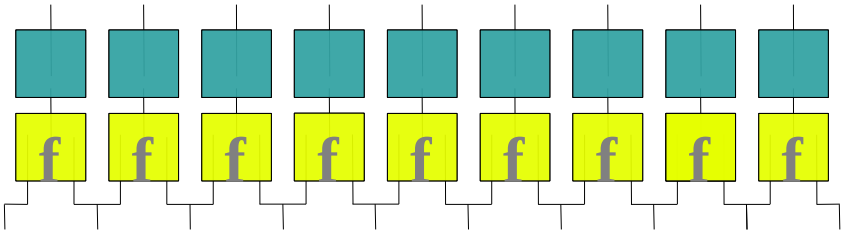


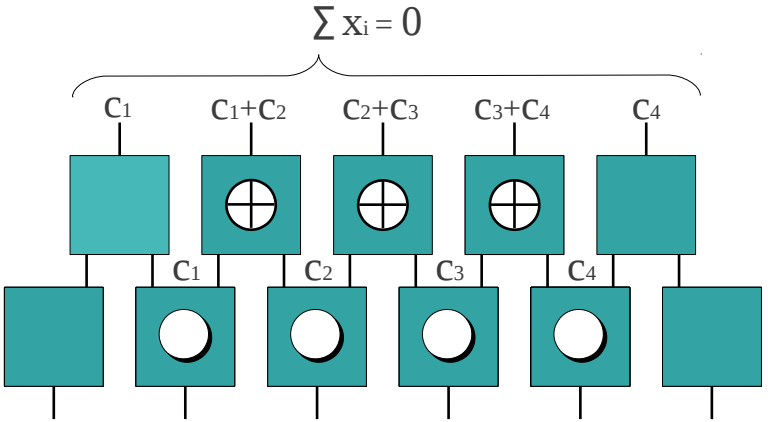


Today for Probabilistic Cellular Automata.
A.k.a what does it mean to be causal in a probabilistic setting?

Traditional definition (Naive PCA)...

- Perform a classical CA
- Per cell noise

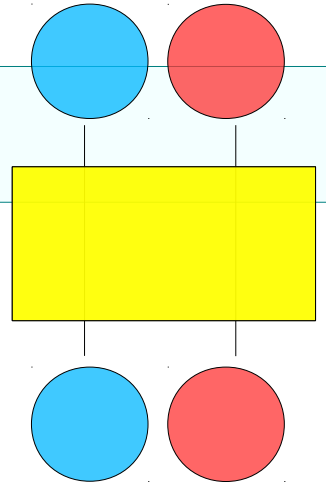


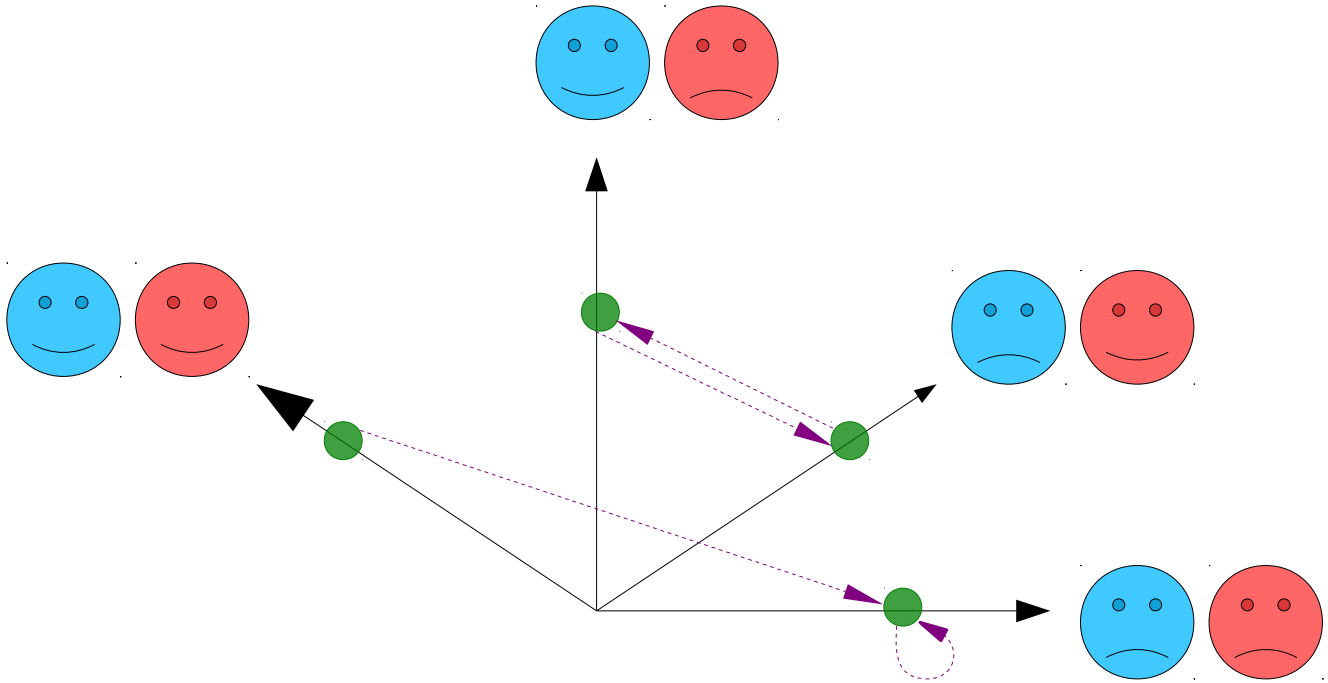


…alas suffers some drawbacks:

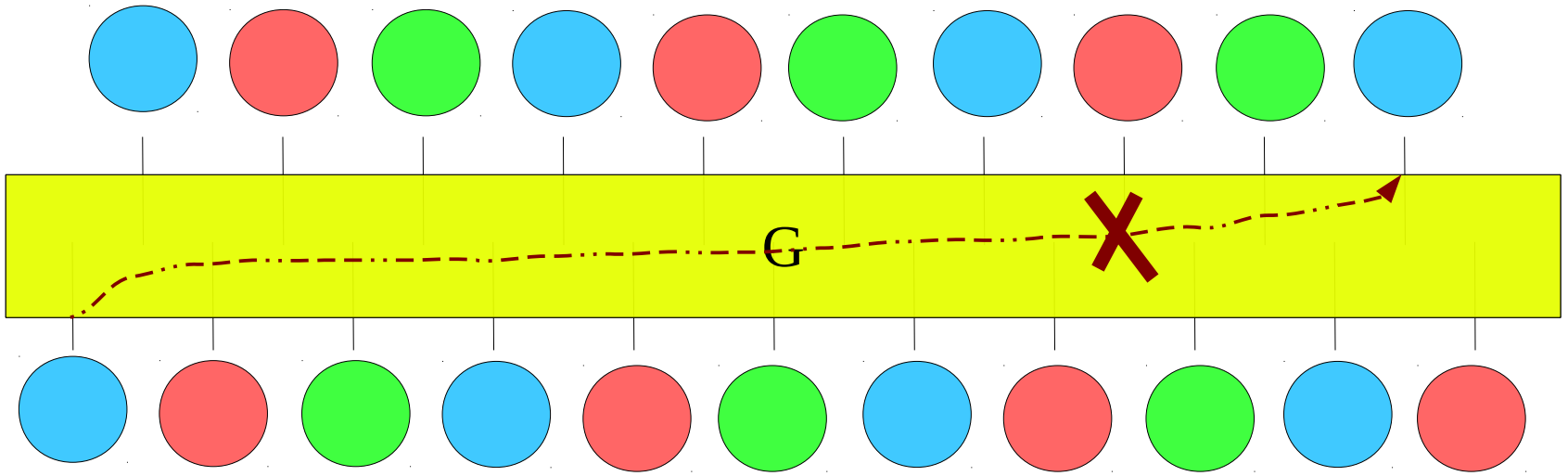
- Incomplete
- Non-composable
- Not based on high-level physical principles (ad hoc?)

But good axiomatizations have been achieved for both Classical and Quantum Cellular Automata. A quick review...





An evolution is described by any function from state space to state space.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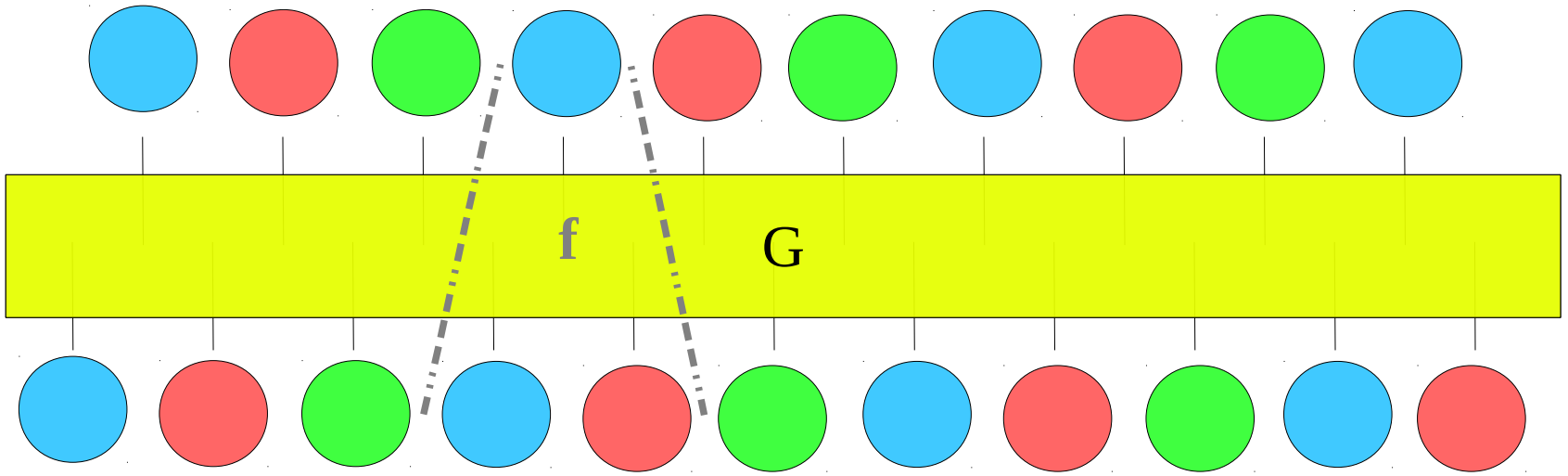
(simple version)



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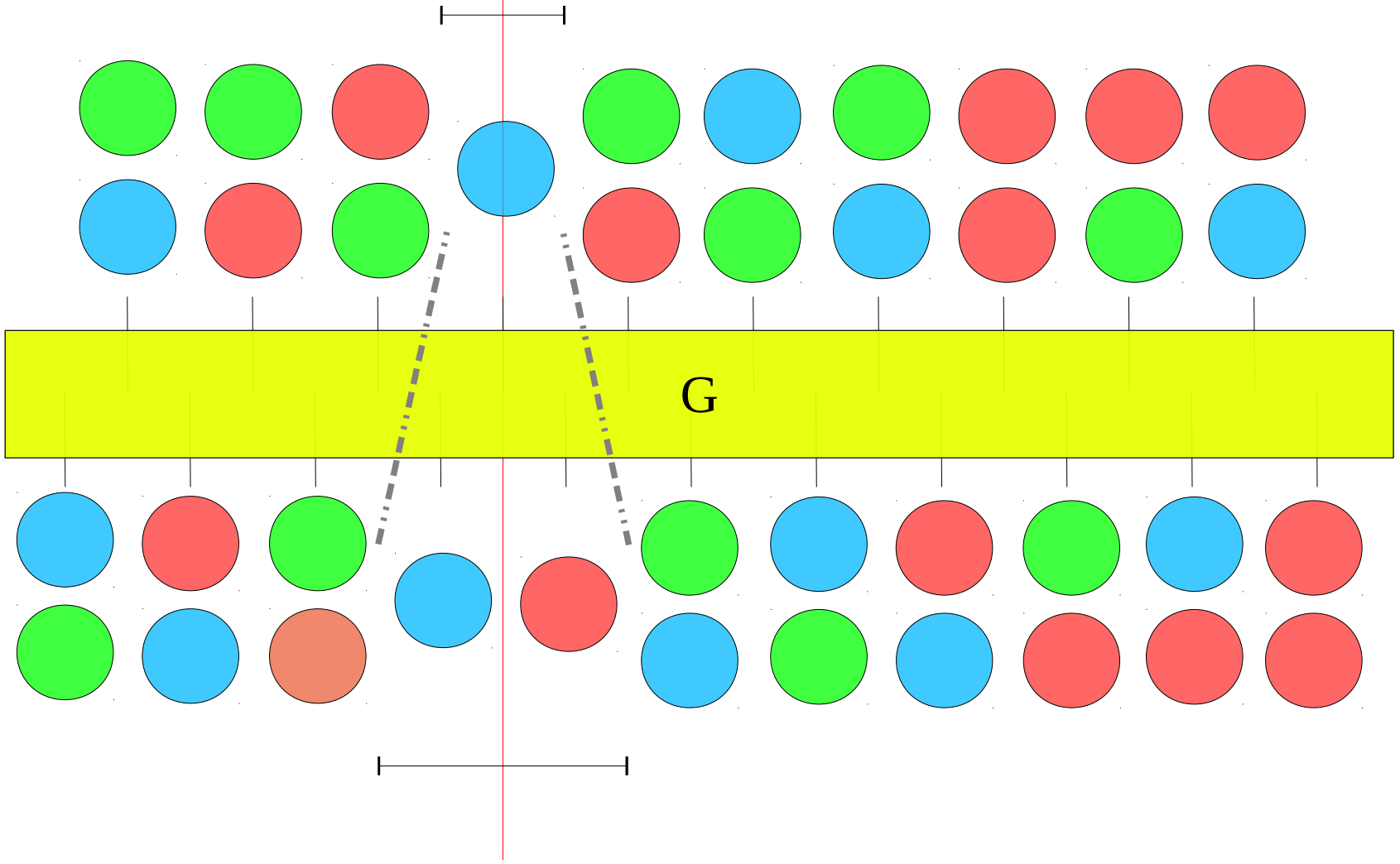

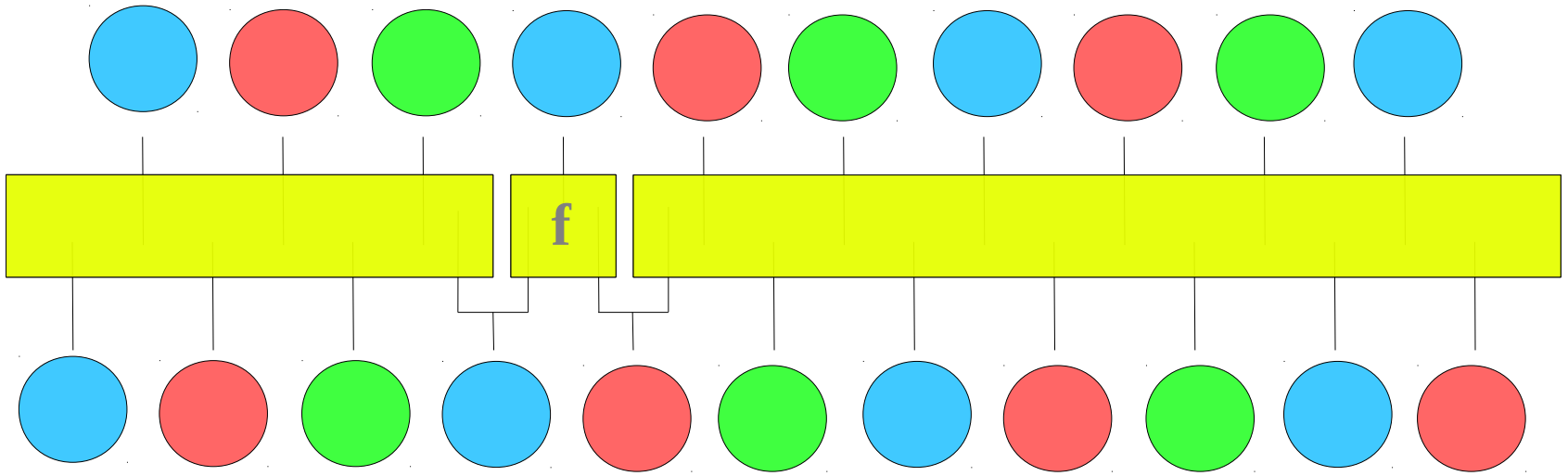
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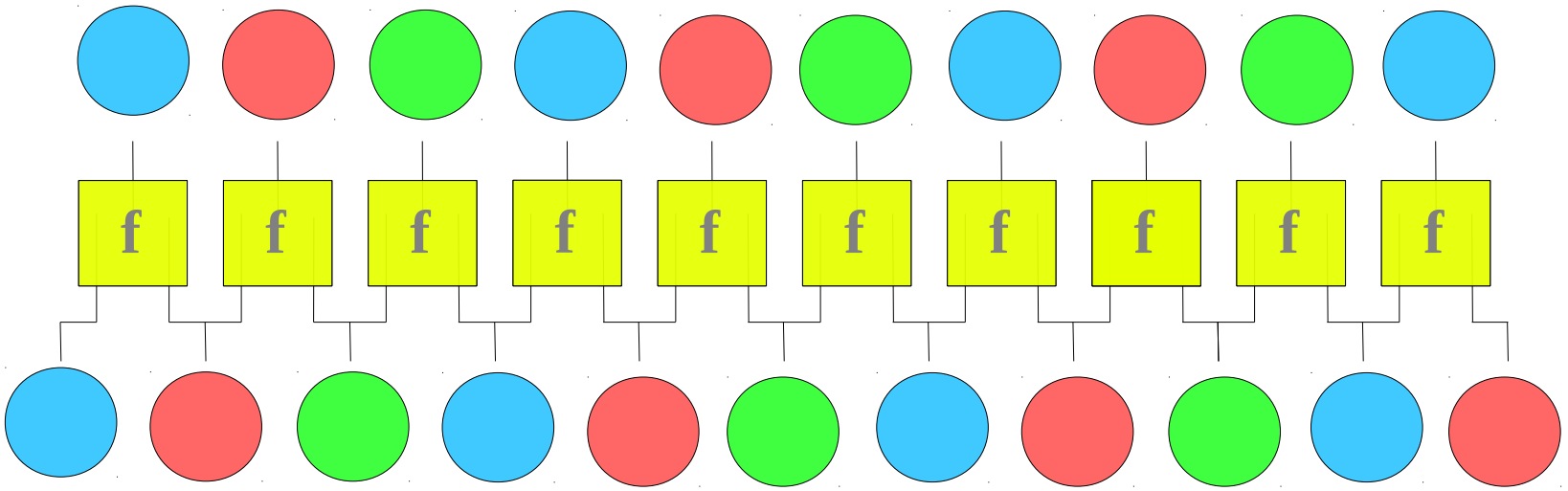
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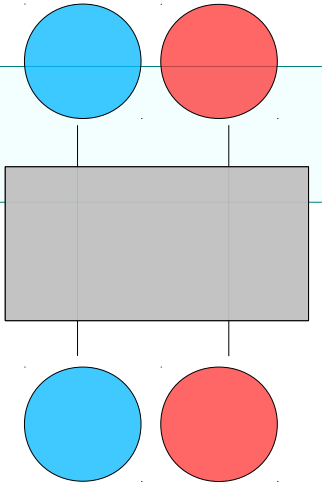
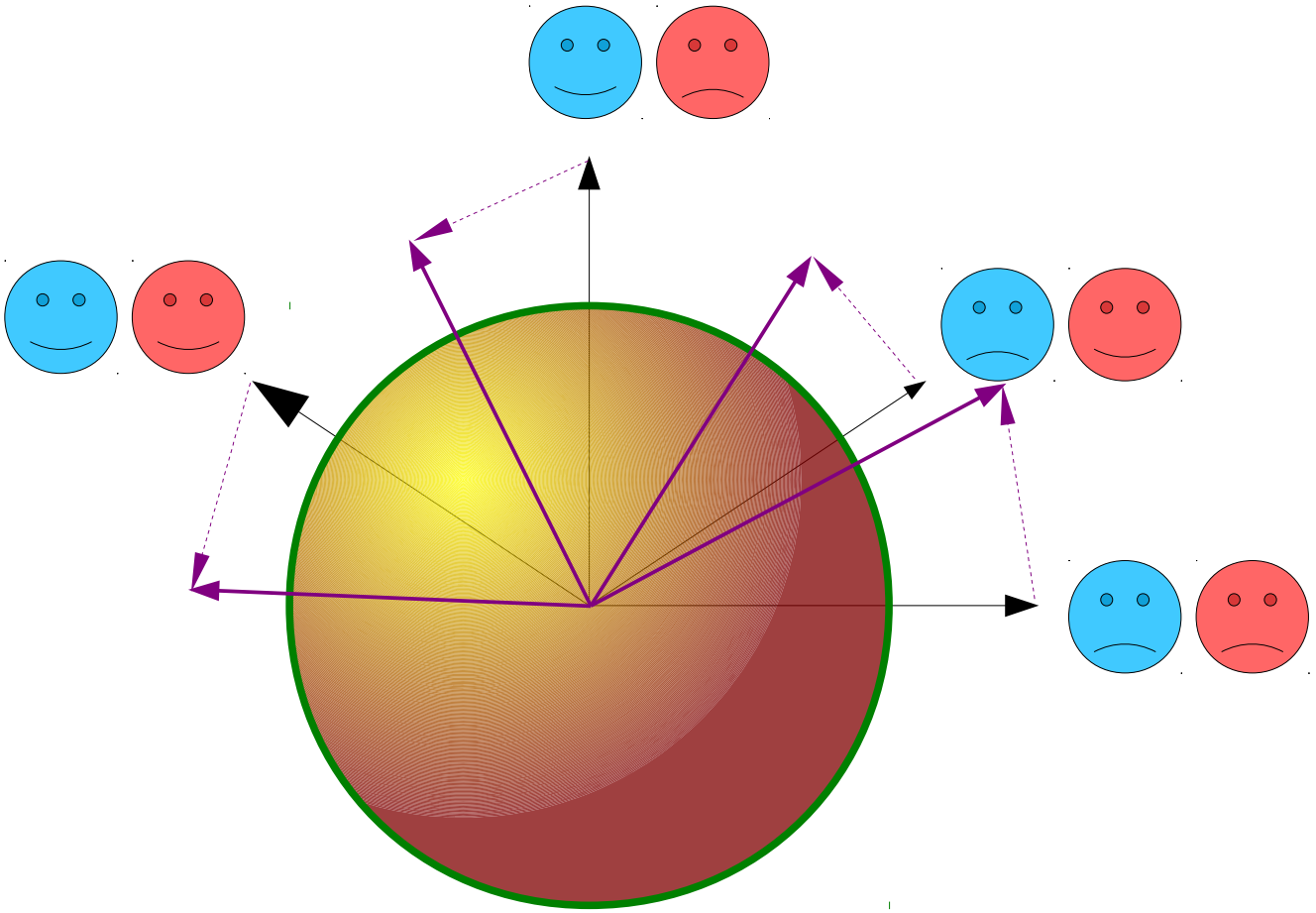
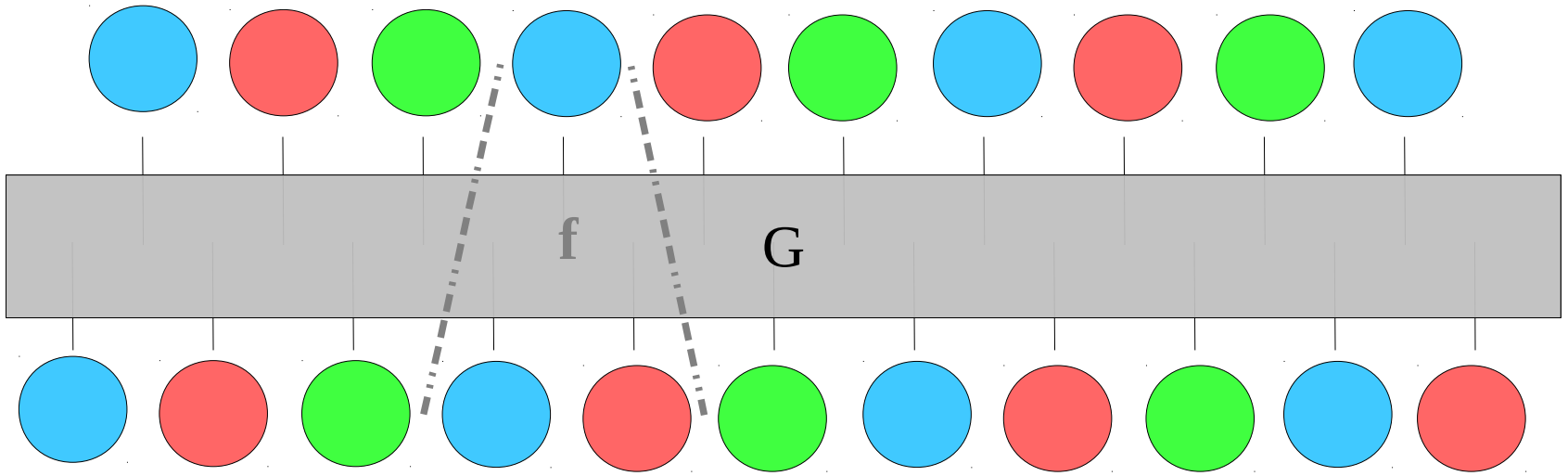




An evolution is described by a linear change of basis in the state space.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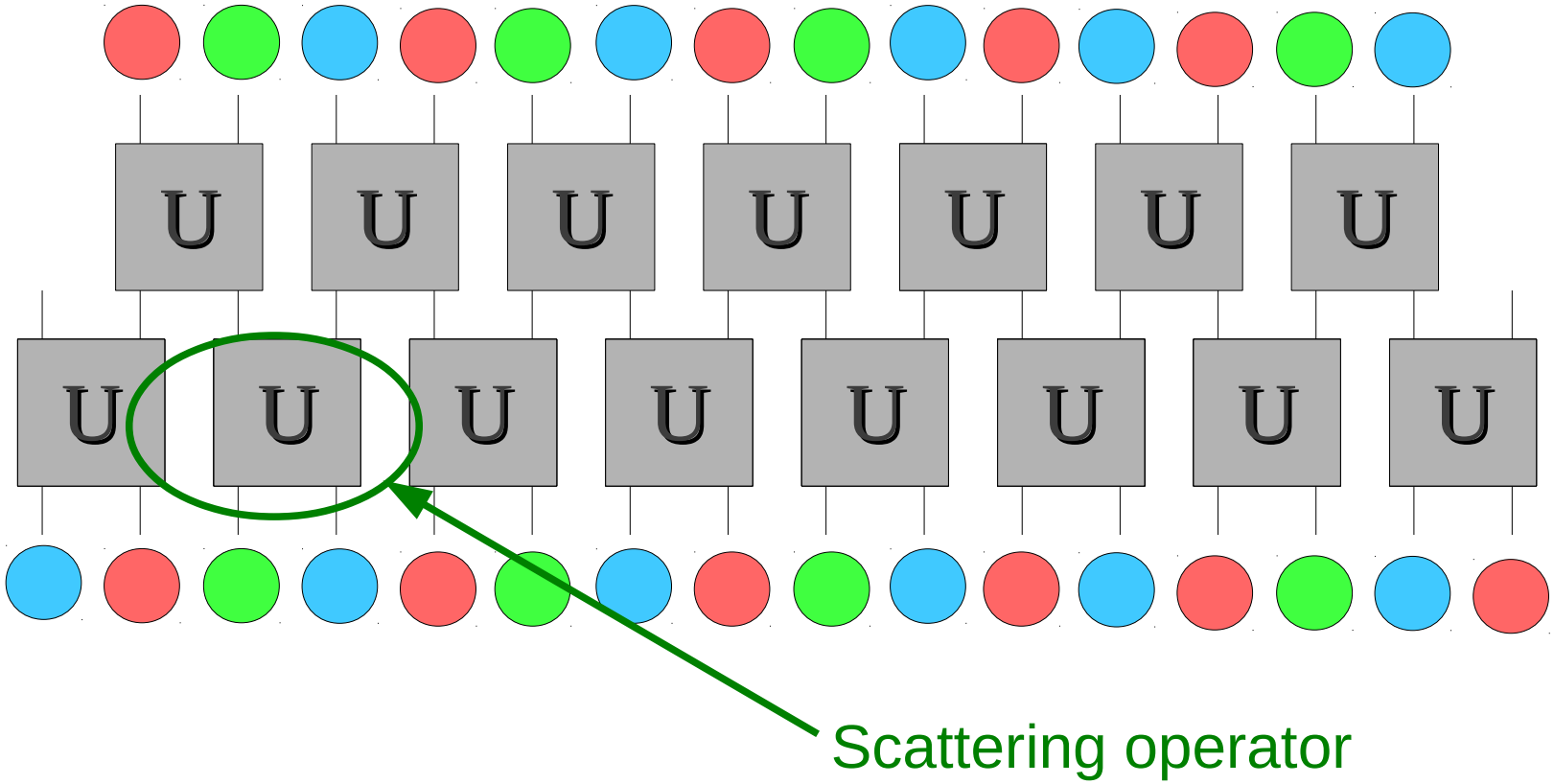
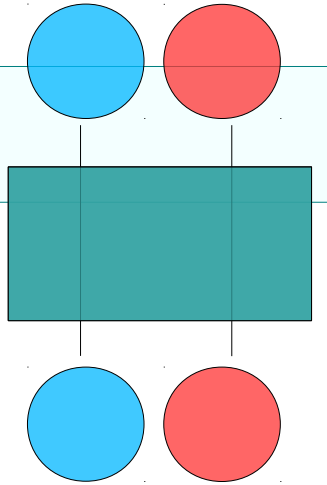
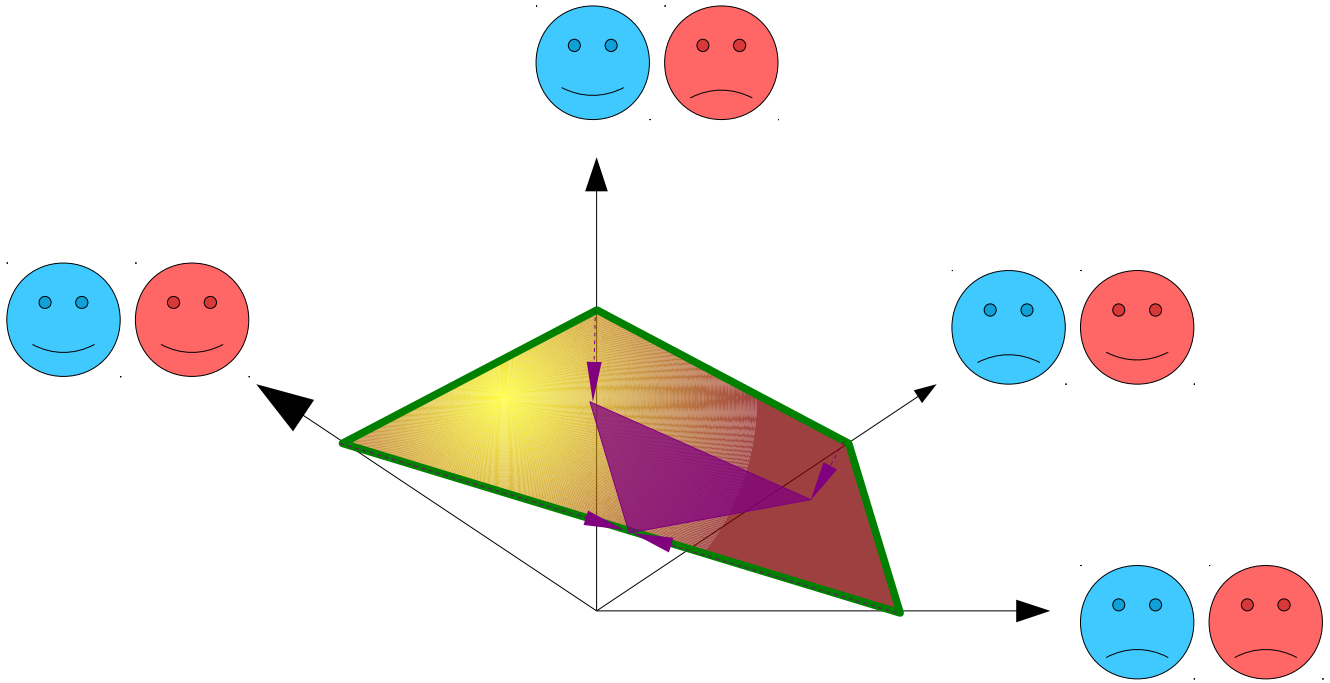
(On thursday 11:00 Room 1!)



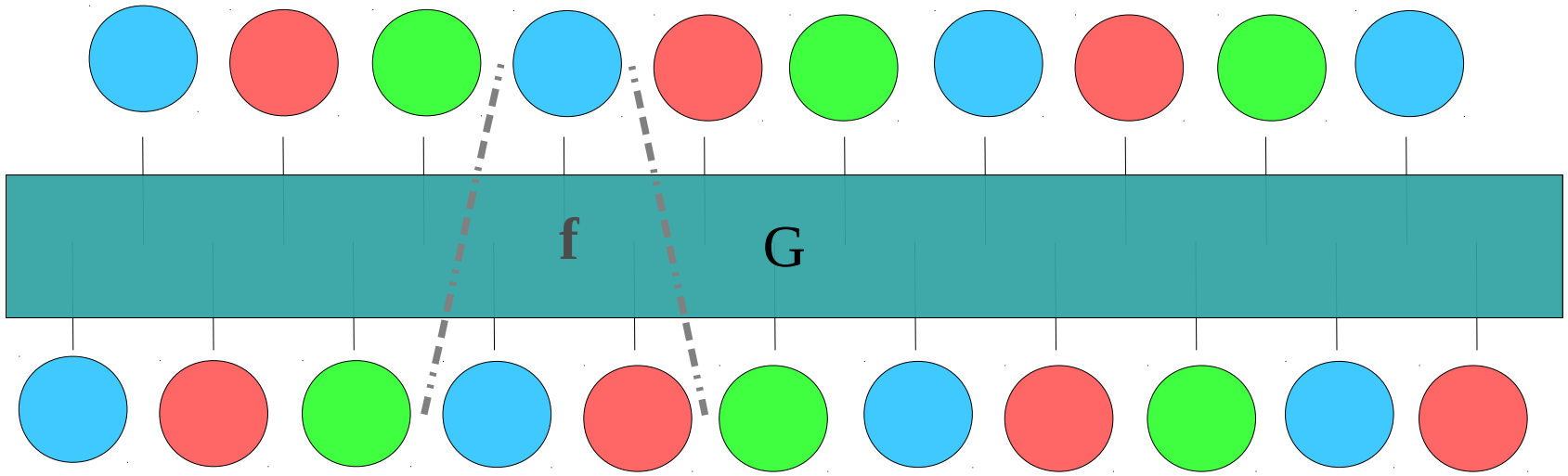
[A., Nesme, Werner, LATA 2008][A., Grattage, Phys. & Comp 2009]

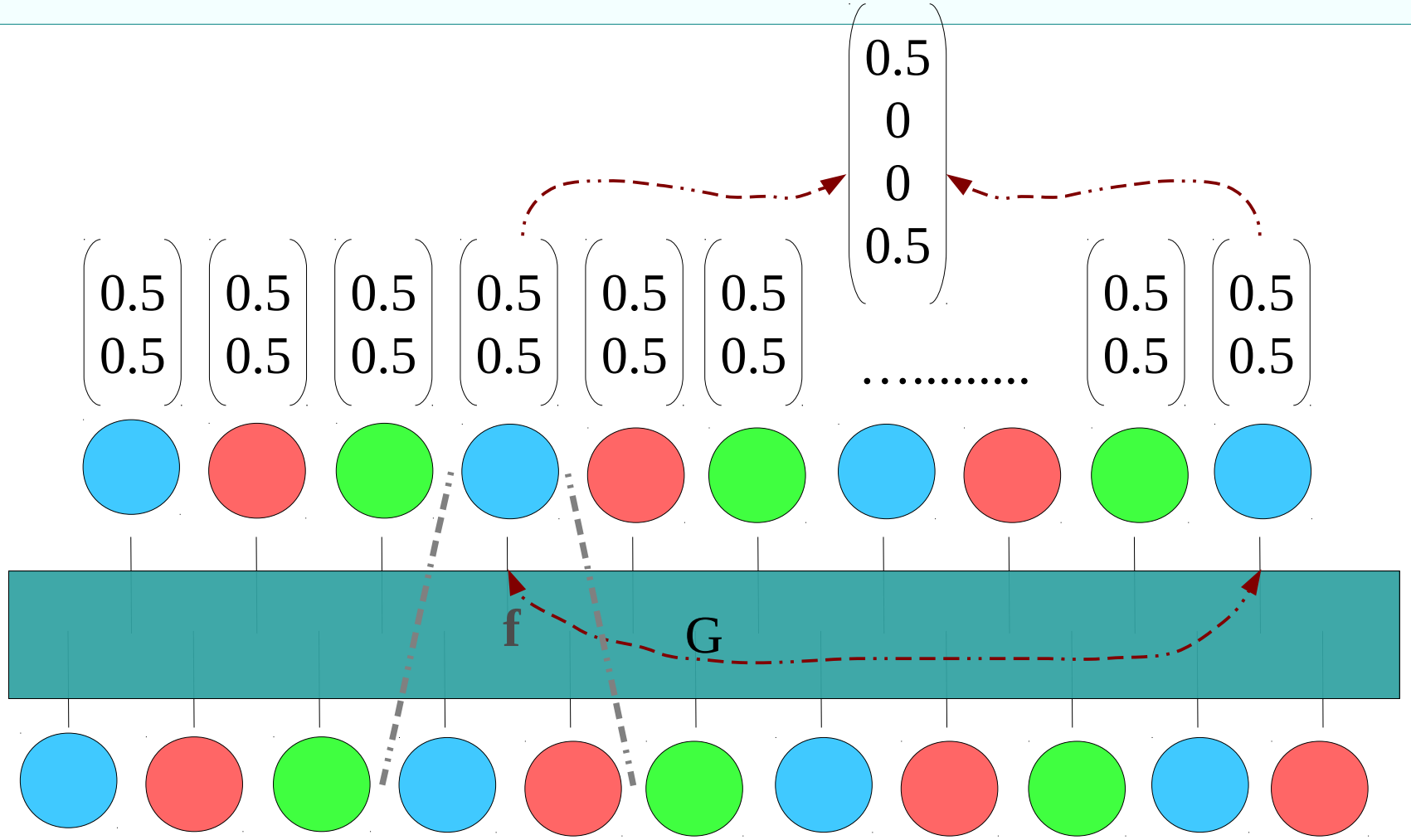




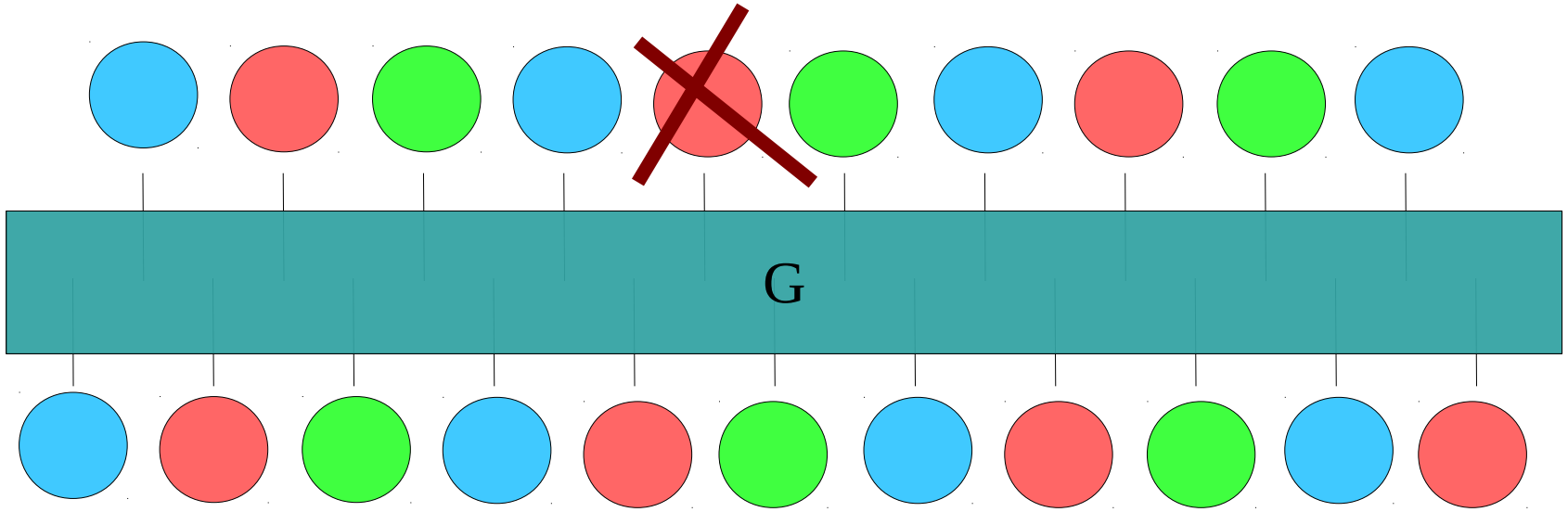
An evolution is described by a linear map from state space to state space.

Probabilistic > Non-signalling evolutions > Definition





An example with f is a constant function:
non-signalling but correlating hence non-localizable!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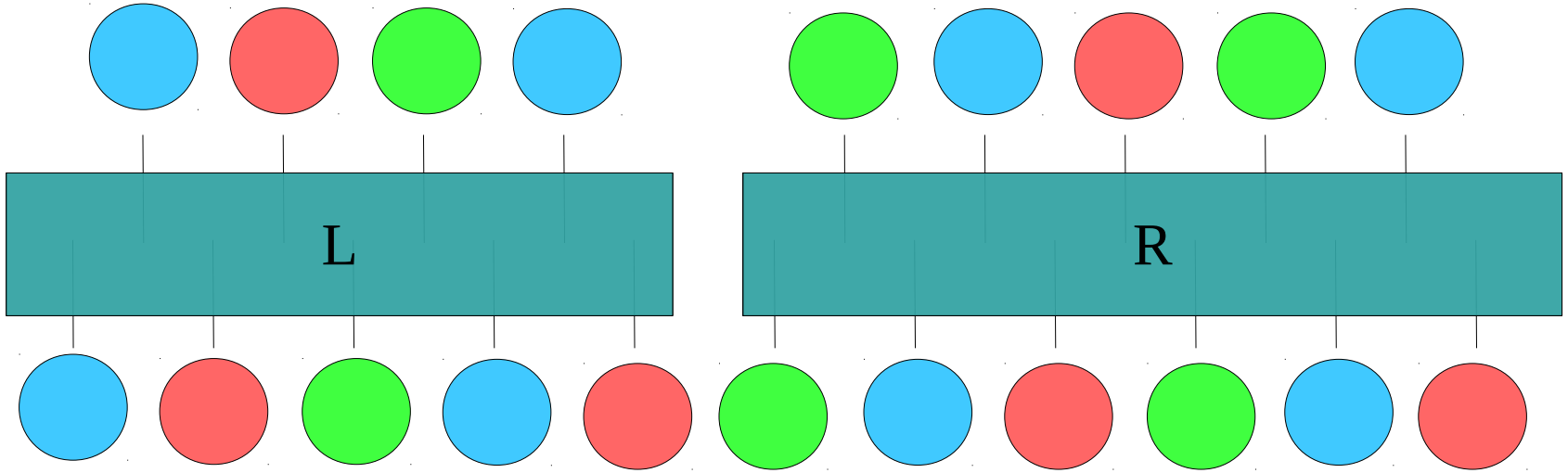
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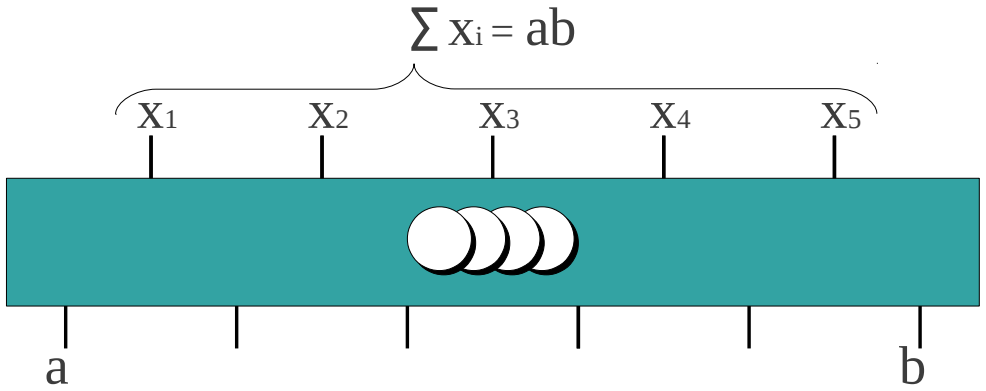
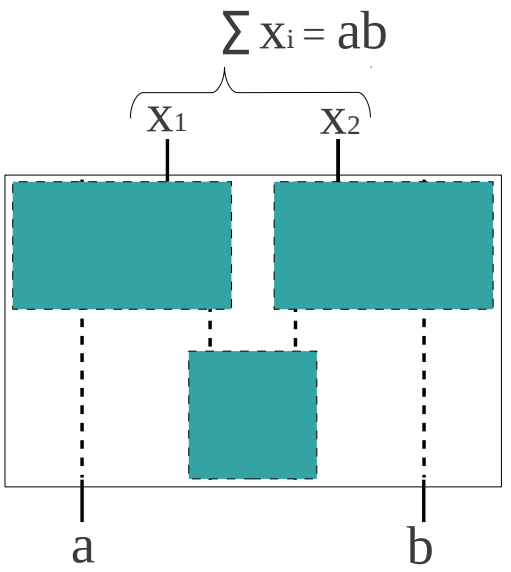
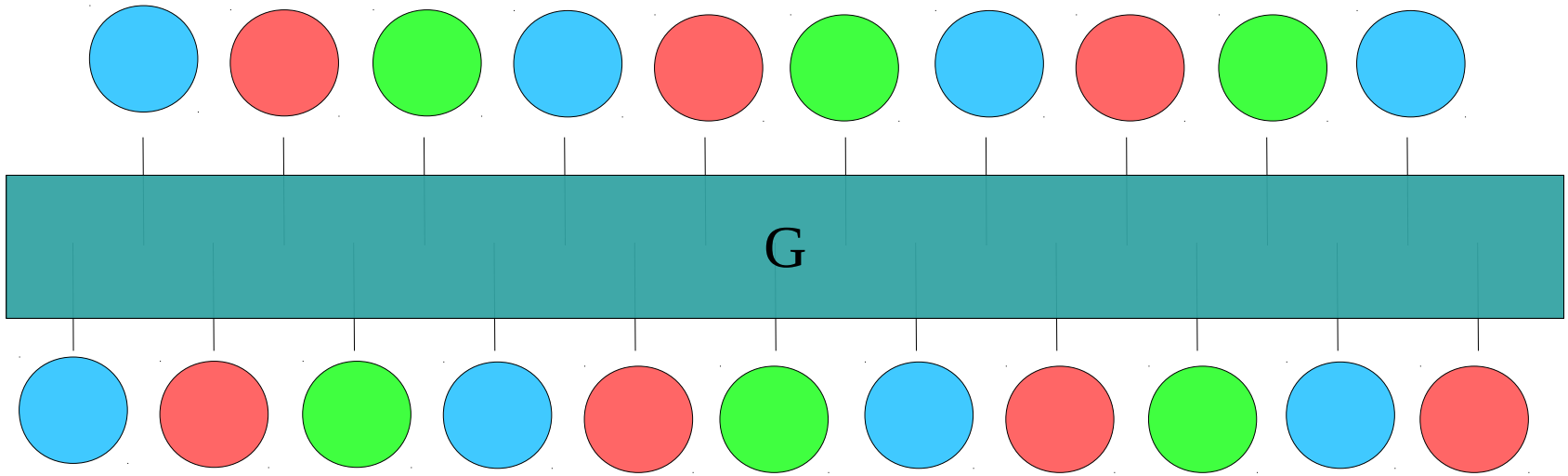
Probabilistic > Non-correlating evolutions > (**Counter-)example**





Non-correlating but not V-causal hence not localizable!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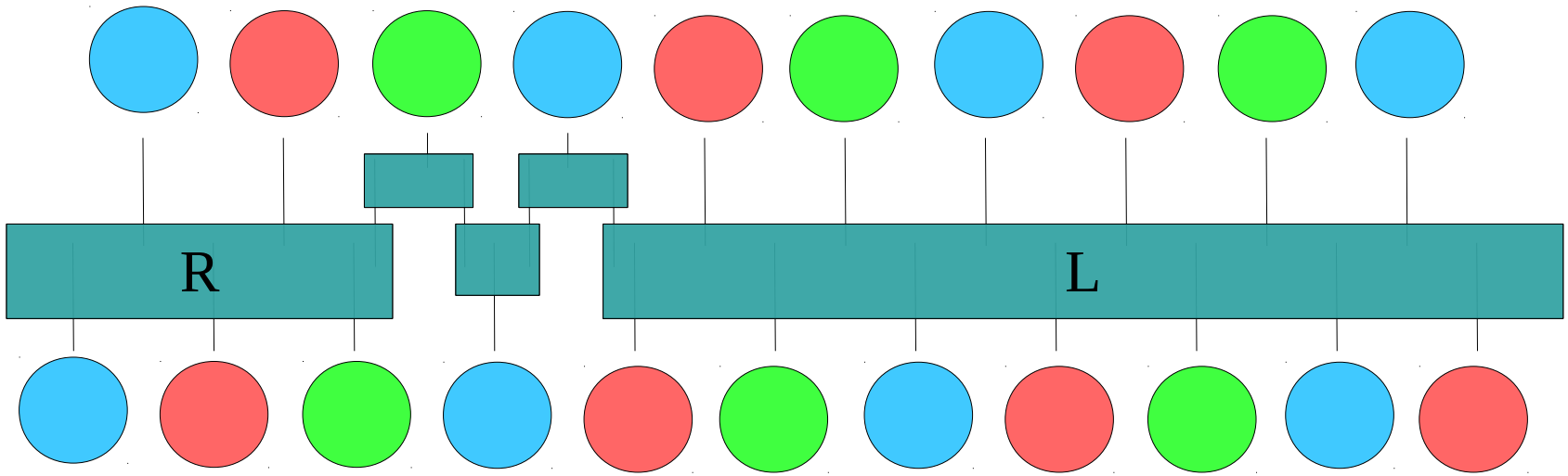
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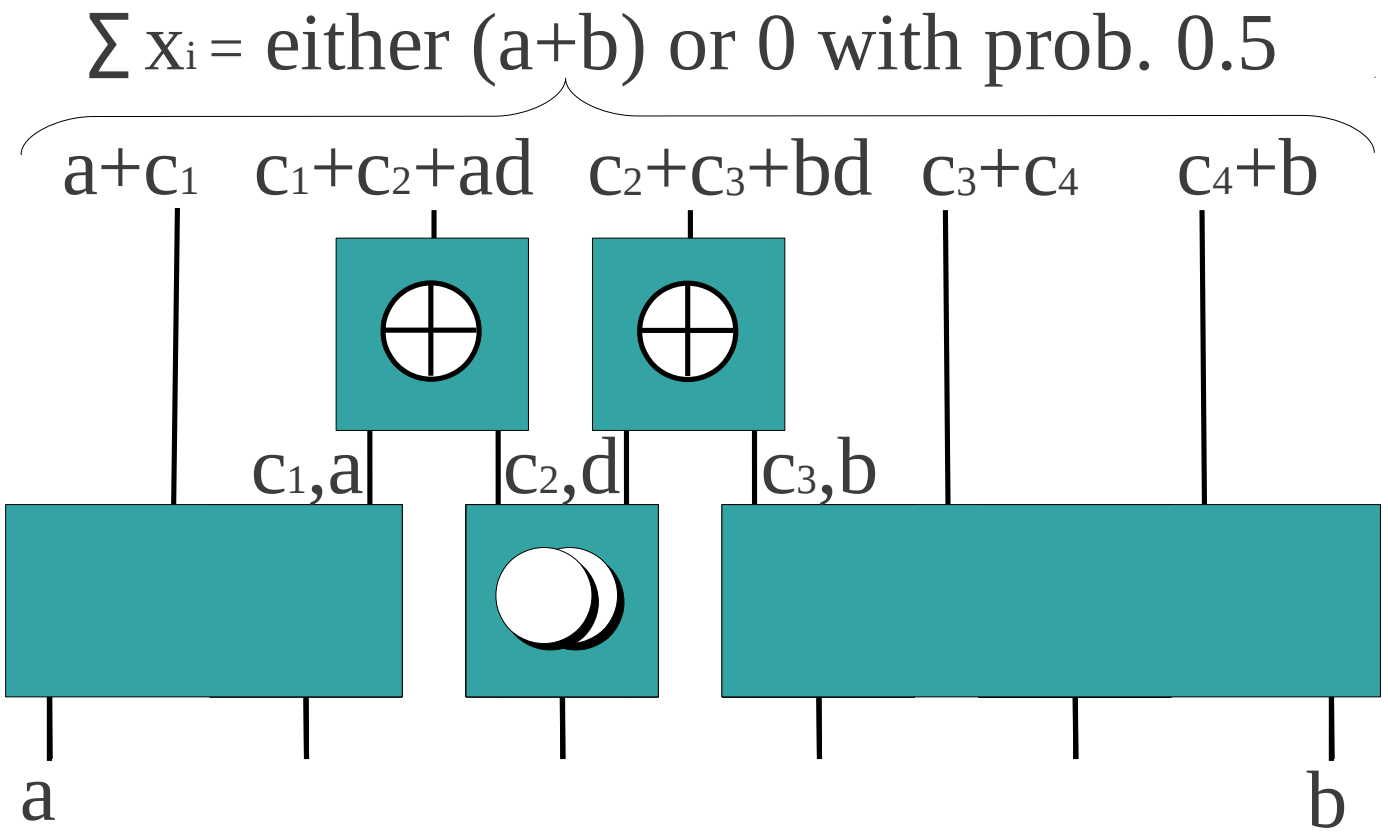
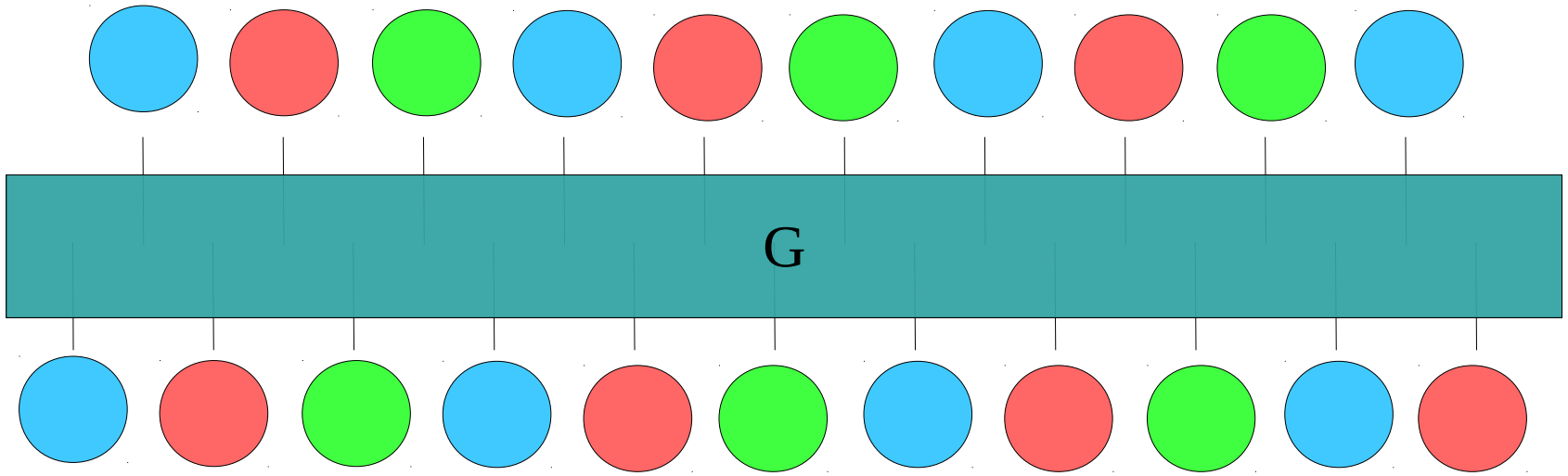


The V-Box:
V-causal but not V²-causal hence not localizable!

Probabilistic > **V²-causal** > **Definition**



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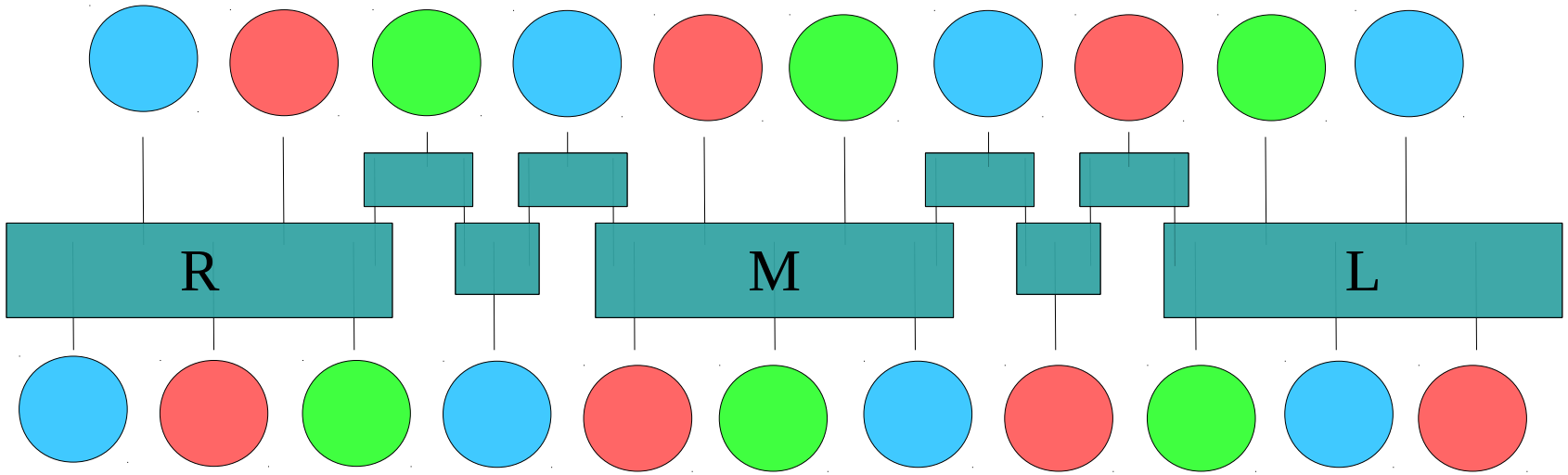
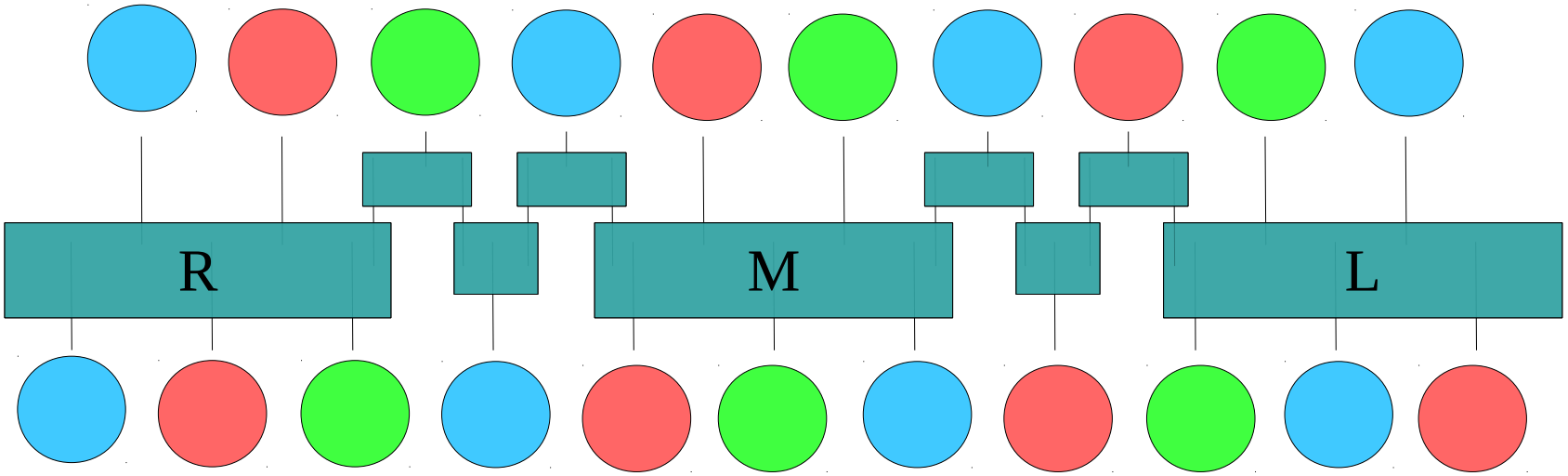


Conjecture:
There exists V^k-Box:
V^k-causal but not V^{k+1}-causal hence not localizable!



Conclusion > **Probabilistic Cellular Automata, definition?**

Traditional definition (Naive PCA)...

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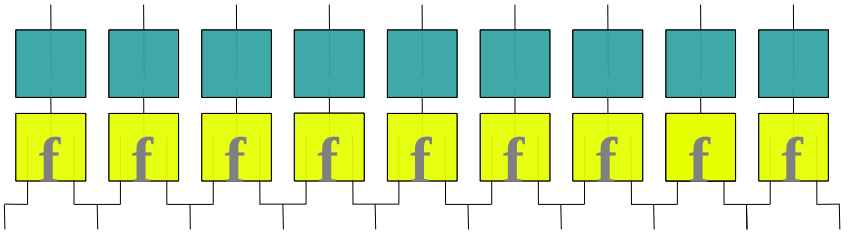
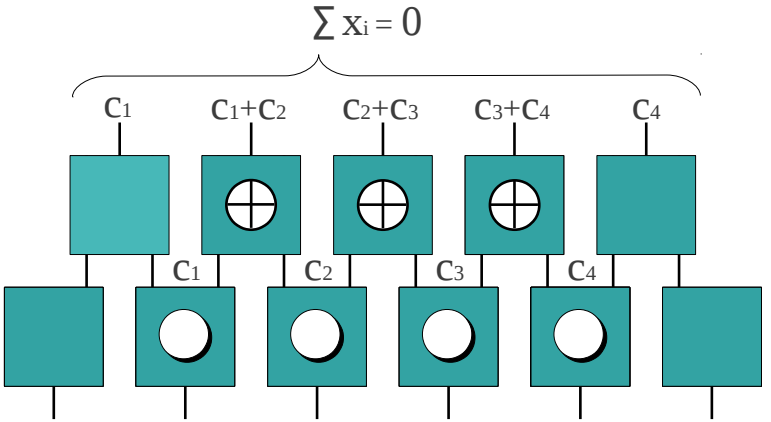
- Per cell noise





…alas suffers some drawbacks:

- Incomplete
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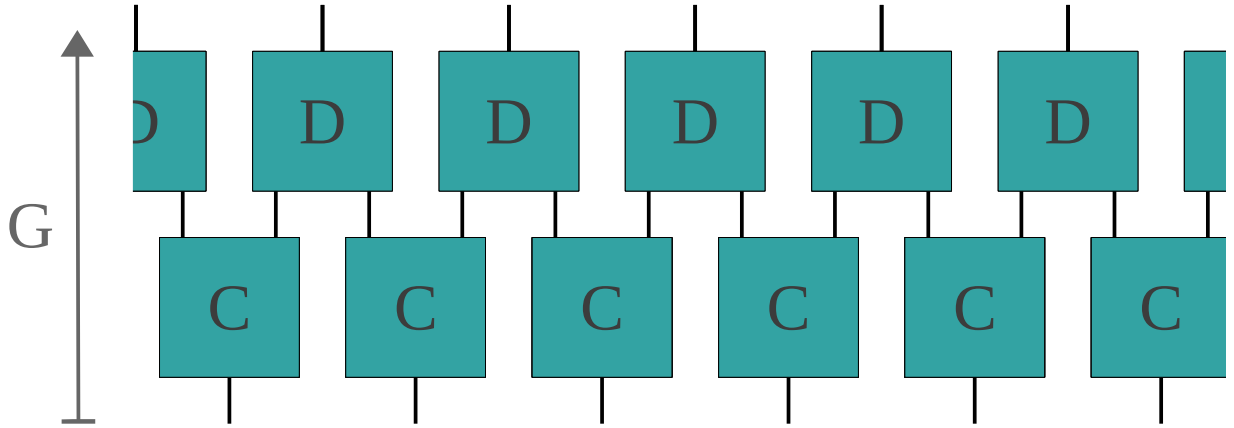
- Not based on high-level physical principles (ad hoc?)

Conclusion > **Probabilistic Cellular Automata, definition?**

Our best definition...

- One-to-two stochastic map
- Two-to-one stochastic map

i.e. V^∞-causality!



…avoids some of the drawbacks:

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- Composable

Conclusion > **Probabilistic Cellular Automata, definition?**

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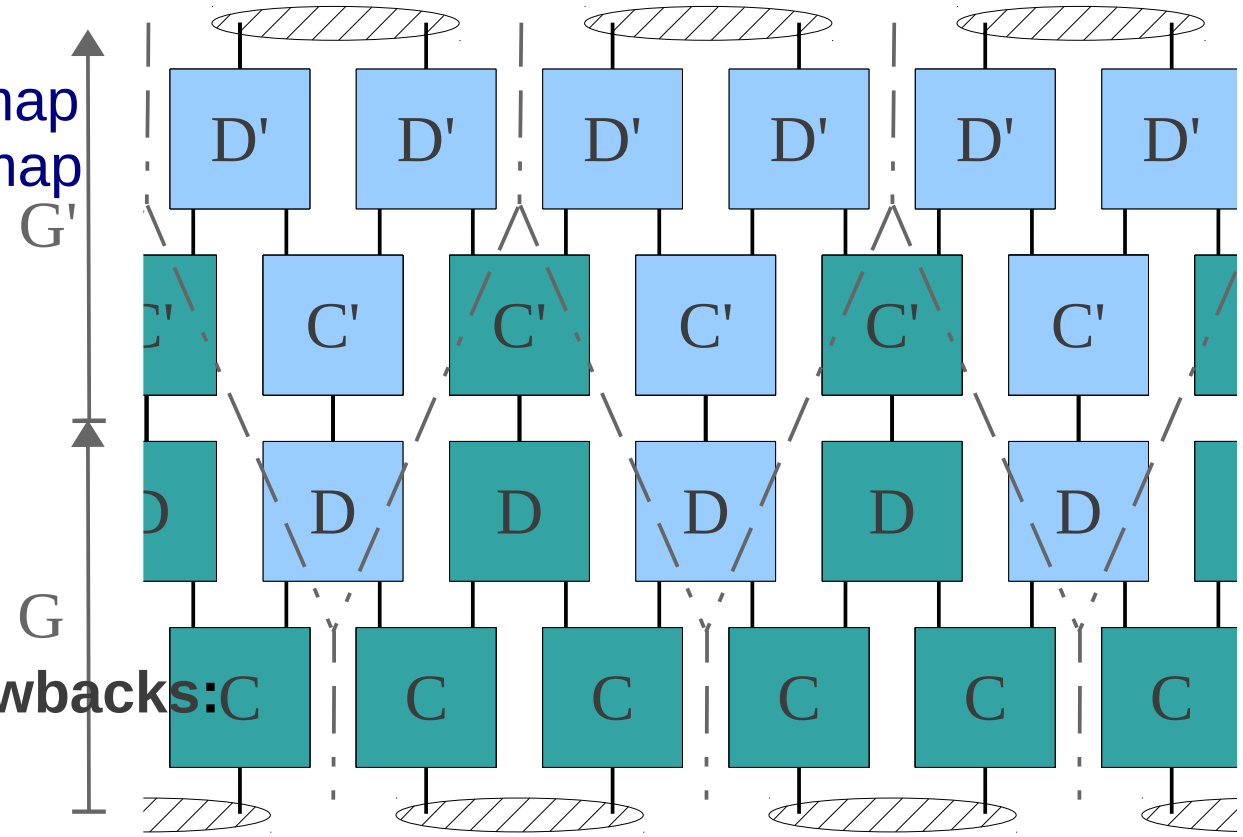

…avoids some of the drawbacks:

- Complete
- Composable
- Based on ~~high-level~~ physical principles

Carries though to open systems Quantum Cellular Automata.
Causality in probabilistic setting is just localizability...