

- XIAOYANG GU, JACK LUTZ, SATYADEV NANDAKUMAR, JAMES ROYER, *Axiomatizing Resource Bounds for Measure*.  
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Resource-bounded measure is a generalization of classical Lebesgue measure that is useful in computational complexity. The central parameter of resource-bounded measure is the *resource bound*  $\Delta$ , which is a class of functions. When  $\Delta$  is unrestricted, resource-bounded measure coincides with classical Lebesgue measure, and when restricted to functions satisfying some complexity constraint, resource-bounded measure imposes internal measure structure on a corresponding complexity class.

Most applications of resource-bounded measure use only the “measure-zero/measure-one fragment” of the theory. For this fragment,  $\Delta$  can be taken to be a class of type-one functions. However, in the full theory of resource-bounded measurability and measure, the resource bound  $\Delta$  also contains type-two functionals. To date, both the full theory and its zero-one fragment have been developed in terms of a list of example resource bounds.

This paper replaces this list-of-examples approach with a careful investigation of the conditions that suffice for a class  $\Delta$  to be a resource bound. Our main theorem says that every class  $\Delta$  that has the closure properties of Mehlhorn’s basic feasible functionals is a resource bound for measure. We also prove that the type-2 versions of the time and space hierarchies that have been extensively used in resource-bounded measure have these closure properties, and moreover, are robust.