The Complexity of Countable Abelian P-Groups

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In descriptive set theory, there is a body of work comparing the difficulty of determining whether certain equivalence relations hold for various classes of structures. The Borel embedding has been a successful formalization for comparing isomorphism for graphs, linear orders, Abelian torsion groups, fields, etc. [FS1989]. Other interesting equivalence relations, such as Turing equivalence and bi-embeddability have also been studied with this notion [Ke1999,Th2010]. In computable structure theory, we would like to determine which classes of structures have *effective* reductions. This notion was studied as the *Turing computable embedding* by Calvert, et al. [CCKM2004,KMV2007]. Quinn characterized which classes Turing computably embed (with respect to isomorphism) in the class of Abelian p-groups of some bounded length, denoted AB^p_α, where α is the bound. The Pull-back Theorem implies that, for a class K to Turing computably embed in AB^p_α, the structures in K must be distinguished by $\Sigma^c_{f(\alpha)}$ sentences, for some f, varying with α [Qu2008]. Motivated by the importance of the Σ^c_α sentences in these embeddings, we define $\mathcal{A} \sim_\alpha \mathcal{B}$ if and only if $\mathcal{A}$ and $\mathcal{B}$ model the same Σ^c_α formulas, and show that proofs of Quinn yield the fact that classes of countable reduced Abelian p-groups of various fixed lengths lie on top relative to our embeddings.

Keywords: computable structure theory, Abelian p-groups, Turing computable embeddings

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