

Characterizing randomness by function tests

Kenshi Miyabe

Kyoto University, Japan
kmiyabe@kurims.kyoto-u.ac.jp

Some recent results say that some randomness notions can be characterized by differentiability of a class of functions. For instance, Brattka, Miller and Nies reproved a result by Demuth, which says that a real is Martin-Löf random if and only if each computable function of bounded variation is differentiable at the real. Brattka et al. also proved that a real is computably random if and only if each nondecreasing computable function on the unit interval is differentiable at the real. Freer, Kjos-Hanssen and Nies showed differentiability of all computable Lipschitz functions characterizes computable randomness. Freer et al. also gave a characterization of Schnorr randomness by differentiability of all Lipschitz functions that are computable in the variation norm.

In general most major randomness notions may be characterized by differentiability of a class of functions. Then we need to ask why a randomness notion corresponds to a notion in analysis.

The integration is closely related to the differentiation and martingales. Hence we study characterizations of some randomness notions using the integration. The author believes that this study will help us to study the further relation between randomness notions and differentiability.

There is another merit of the use of the integration. A characterization using the integration may be generalized easily. Indeed we give some characterizations over a general space.

In this talk we study the following three randomness notions: Martin-Löf randomness, Schnorr randomness and Kurtz randomness. We generalize the definitions to over a general space and characterize them by tests using the integration. We call the tests *function tests*.

A function test is essential the same as a uniform test. A uniform test is a function from a pair of a measure and a point. In contrast we consider a function from a point with a fixed measure. Hence we prefer to call it a function test.

A function test is also a kind of a martingale. In this kind of view, it is natural for us to be able to characterize many randomness notions.