

The Bolzano-Weierstraß principle, the cohesive principle and program extraction

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Let BW be the usual Bolzano-Weierstraß principle, i.e. the statement that each bounded sequence of real numbers contains a subsequence converging at the rate 2^{-n} . Let BW_{weak} be the statement that each bounded sequence of reals contains a —possibly slowly— converging subsequence, i.e. a subsequence converging but possibly without any computable rate of convergence.

We show that BW is instancewise equivalent to WKL for Σ_1^0 -trees and that BW_{weak} is instancewise equivalent to the cohesive principle. Thus the degrees $d \gg 0'$ (i.e. the degrees d that contain an infinite branch of each $0'$ -computable 0/1-tree) are exactly those degrees that contain for each computable sequence (x_n) a subsequence converging at the rate 2^{-n} . In particular, there is a degree d that is low over $0'$ that contains a solution of each computable instance of BW. Using the classification of the cohesive principle of Jockusch and Stephan one obtains that a slowly converging subsequence of (x_n) is computable in a degree d that is low_2 , i.e. $d'' = 0''$, and thus that BW_{weak} does not compute $0'$.

We further comment on program extraction results we obtained (jointly with Ulrich Kohlenbach) for principles that are weaker than Ramsey's theorem for pair like the cohesive principle.

References

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2. Kreuzer, Alexander P. and Kohlenbach, Ulrich, *Term extraction and Ramsey's theorem for pairs*. Submitted, preprint available at <http://www.mathematik.tu-darmstadt.de/~akreuzer/files/TermExtractionAndRT22.rev.pdf>