Rigidity of the uppersemilattice of the enumeration degrees

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In 1966 Sacks and in 1967 Rogers stated the basic question about Turing degrees if there are any nontrivial automorphisms in the uppersemilattice of T-degrees. In case that there aren't nontrivial automorphisms in some uppersemilattice we say it is rigid. The same question was stated about other degrees, including enumeration degrees (e-degrees).

In 1977 Jockusch and Solovay [1] and in 1979 Richter [2] and Epstein [3] proved that every automorphism is the identity on the cone above $0^{(3)}$ for Turing degrees. In 1986 Slaman and Woodin [4] approved the above result showing that every automorphism is the identity on the cone above $0''$. Using the connections between both T- and e-jumps in [5] it was obtained that every automorphism is the identity on the cone above $0^{(4)}$ for e-degrees.

In [6] there have been considered only unary partial structures without equality and it's been given necessary and sufficient conditions for a partial structure to have an e-degree, i.e. to have a least enumeration. Those structures "code" the whole its information in their e-degrees. One of the main reasons to be considered e-degrees of unary structures was the problem of rigidity of e-degrees.

In the present work it is proved the following

Theorem 1. *The upper semi-lattice $\mathfrak{D}_e^* = \langle D_e^*, \leq \rangle$ of all e-degree with the relation $\leq$ is rigid.*

References

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