Decidable and Undecidable Problems Related to Complexity Analysis of Loop Programs

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We investigate the decidability of the *feasibility problem* for imperative programs with bounded loops. A program is called feasible if all values it computes are polynomially bounded in terms of the input. The feasibility problem is representative of a group of related properties, like that of polynomial time complexity. It is well known that such properties are undecidable for a Turing-complete programming language. They may be decidable, however, for fragmentary languages, that are Turing-incomplete. But if such fragments are expressive enough, they do pose a challenge for analysis. We are interested in tracing the edge of decidability for the feasibility problem and similar problems.

In this talk we will pose some open problems and give a summary of results published in [BJK08], [BA10] and [BK1?].

We consider programs where only indefinite loops are allowed, and programs where only definite loops are allowed:

- *indefinite* loops `?loop X {C}` are loops where the body `C` will be executed an arbitrary (nondeterministic) number of times, but the variable `X` gives an upper bound on the number of iterations.
- *definite* loops `!loop X {C}` are deterministic loops where `X` gives the exact number of times `C` will be executed.

A second language feature that we vary, is the kind of assignment statements. We consider

- *ordinary* assignments `X:=`exp where the variable `X` is assigned the exact value given by the expression exp.
- *weak* assignments `X` $:\leq exp$ where `X` is assigned any value bounded by exp
- *max* assignments `X:=`$^{\max} exp$ where `X` never decreases its value (`X` keeps its old value if this value is greater than the one given by exp).

The table below gives a summary of our results.

constants: none		0	0, 1
indefinite loops all types of assignments	PTIME	PSPACE	?
definite loops max assignments	PTIME	PTIME	?
definite loops weak/standard ass	undecidable	undecidable	undecidable

References

[BA10] Amir M. Ben-Amram. On decidable growth-rate properties of imperative programs. In Patrick Baillot, editor, *International Workshop on Developments in Implicit Computational complExity (DICE 2010)*, volume 23 of *EPTCS*, pages 1–14. ArXiv.org, 2010.

[BJK08] Amir M. Ben-Amram, Neil D. Jones, and Lars Kristiansen. Linear, polynomial or exponential? complexity inference in polynomial time. In Arnold Beckmann, Costas Dimitracopoulos, and Benedikt Löwe, editors, *Logic and Theory of Algorithms, Fourth Conference on Computability in Europe, CiE 2008*, volume 5028 of *LNCS*, pages 67–76. Springer, 2008.

[BK1?] Amir M. Ben-Amram and Lars Kristiansen. On the Edge of Decidability in Complexity Analysis of Loop Programs. Submitted.