

- MARGARITA KOROVINA, OLEG KUDINOV, *A Finite Language for Computable Metric Spaces.*

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The main goal of the research presented in this paper is to provide a logical framework for studying computability over computable metric spaces. We consider the following problem. Given a computable metric space is there a finite predicate language such that effective openness and computability could be characterised by appropriate formulas? In order to attack this problem we represent a computable metric space as a positive predicate structure in a finite language and employ Σ -definability theory. In this settings we prove that for every computable metric space there exists a finite predicate language such that the following statements hold. A subset of a computable metric space is effectively open if and only if it is Σ -definable. The computable functions coincide with the effectively continuous functions in the topology generated by existential formulas. The existential theory of a computable metric space is computable enumerable. A total real-valued function is computable if and only if the epigraph and the hypograph are Σ -definable.