The First-order Fragments of Second-order Theories

Theodore A. Slaman
Department of Mathematics
University of California Berkeley
Berkeley, CA 94720-3840 USA

We will discuss the question of what number-theoretic theoretic consequences follow from infinitary hypotheses. The best known example is Harrington's Conservation Theorem which asserts that if a Π^1_1-statement φ can be proven from the hypothesis that 2^ω is compact, then it can be proven without that assumption. Technically, if $WKL_0 \vdash \varphi$, then $RCA_0 \vdash \varphi$. In the more restrictive case that φ an arithmetic sentence, if $WKL_0 \vdash \varphi$, then $P^- + I\Sigma_1 \vdash \varphi$. We review the prerequisites and then discuss the known conservation and non-conservation theorems for the infinitary principles involving measure/randomness, category/genericity, and combinatorics/homogeneity.